1. INTRODUCTION

Matrices provide a convenient method of arranging numbers, symbols, or mathematical expressions in rows and columns. They are useful for representing information in a compact and organized form and are applied in mathematics, science, engineering, economics, and computer applications.

2. DEFINITION OF A MATRIX

A matrix is a rectangular arrangement of numbers, symbols, or expressions in rows and columns. It is generally represented by a capital letter such as A, B, C, P, or Q. The individual entries are called elements or entries.

$$A = \begin{bmatrix} a_{11}\, a_{12}\, a_{13} \cdots a_{1j} & \cdots a_{1n} \\ a_{21}\, a_{22}\, a_{23} \cdots a_{2j} & \cdots a_{2n} \\ a_{31}\, a_{32}\, a_{33} \cdots a_{3j} & \cdots a_{3n} \\ a_{i1}\, a_{i2}\, a_{i3} \cdots\, a_{ij} & \cdots a_{in} \\ a_{m1}\, a_{m2}\, a_{m3} \cdots a_{mj} & \cdots a_{mn} \end{bmatrix}$$

3. ORDER OF A MATRIX

If a matrix has m rows & n columns, its order is m × n.
A general matrix is written as $A = [a_{ij}]_{m\times n}$, where a_{ij} is the element in the i^{th} row and j^{th} column.

4. TYPES OF MATRICES

Row matrix: one row.

Column matrix: one column.

Rectangular matrix: number of rows is not equal to number of columns.

Square matrix: number of rows equals number of columns.

Zero matrix: every element is zero.

Diagonal matrix: a square matrix in which all non-diagonal elements are zero.

Scalar matrix: a diagonal matrix with equal diagonal elements.

Unit/Identity matrix: a diagonal matrix with every diagonal element equal to 1.
It satisfies AI = IA = A.

Trigonal matrix : A triangular matrix is a special type of square matrix featuring zeros either above or below the main diagonal.

Types of Matrix

1. Row Matrix	2. Column Matrix	3. Rectangular Matrix
$(1\ 2\ 3)$	$\begin{bmatrix}1\\2\\3\end{bmatrix}$	$\begin{bmatrix}1&3&4\\2&5&2\end{bmatrix}$
4. Square Matrix	**5. Zero Matrix**	**6. Diagonal Matrix**
$\begin{bmatrix}1&3&4\\5&2&4\\1&9&6\end{bmatrix}$	$\begin{bmatrix}0&0&0\\0&0&0\\0&0&0\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&2&0\\0&0&3\end{bmatrix}$
7. Scalar Matrix	**8. Unit Matrix**	**9. Upper and lower triangular matrix**
$\begin{bmatrix}2&0&0\\0&2&0\\0&0&2\end{bmatrix}$	$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$	$\begin{bmatrix}1&0&0\\2&4&0\\3&5&6\end{bmatrix}\begin{bmatrix}5&8&3\\0&1&2\\0&0&7\end{bmatrix}$

5. EQUALITY OF MATRICES

Two matrices A and B are equal only when they have the same order and all corresponding elements are equal. Thus, A = B only when aij = bij for every corresponding position.

6. TRANSPOSE OF A MATRIX

The transpose of a matrix is obtained by changing rows into columns and columns into rows. It is denoted by A^T or A’.

Important properties are:

→ $(A^T)^T = A$
→ $(A+B)^T = A^T + B^T$
→ $(A-B)^T = A^T - B^T$
→ $(AB)^T = B^T A^T$

7. SYMMETRIC MATRIX

A square matrix A is symmetric if $A^T = A$. Therefore, $a_{ij} = a_{ji}$; elements on opposite sides of the main diagonal are equal.

8. SKEW-SYMMETRIC MATRIX

A square matrix A is skew-symmetric if $A^T = -A$. Therefore, $a_{ij} = -a_{ji}$. All diagonal elements of a skew-symmetric matrix are zero.

9. ADDITION OF MATRICES

Two matrices can be added only when they have the same order.

Corresponding elements are added: $A+B = [a_{ij}+b_{ij}]$.

Properties:

- → $A+B=B+A$
- → $A+(B+C)=(A+B)+C$
- → $A+O=A$;
- → $A+(-A)=O$.

10. SUBTRACTION OF MATRICES

Two matrices can be subtracted only when they have the same order. Corresponding elements are subtracted: $A-B=[a_{ij}-b_{ij}]$. In general, $A-B$ is not equal to $B-A$.

11. SCALAR MULTIPLICATION

When every element of a matrix is multiplied by a number k, the operation is called scalar multiplication.

Important properties are :

- → $k(A+B)=kA+kB$
- → $(k+l)A=kA+lA$
- → $k(lA)=(kl)A$.

12. MULTIPLICATION OF MATRICES

If A is of order $m \times n$ and B is of order $p \times q$, the product AB exists only when $n=p$. The product then has order $m \times q$. Each entry is found by multiplying a row of the first matrix by a column of the second matrix and adding the products.

PROPERTIES OF MATRIX MULTIPLICATION:

- → Matrix multiplication is associative: $(AB)C=A(BC)$.
- → It is distributive: $A(B+C)=AB+AC$ and $(A+B)C=AC+BC$.
- → The identity property is $AI=IA=A$.
- → Matrix multiplication is generally not commutative, so $AB \neq BA$.
- → It is also possible for non-zero matrices A and B to have $AB=O$.

13. INVERTIBLE MATRIX

A square matrix A is invertible if there is a square matrix B such that $AB=BA=I$. The matrix B is called the inverse and is denoted by A^{-1} . Hence $AA^{-1}=A^{-1}A=I$. An invertible matrix has a unique inverse.

For a non-singular matrix, $A^{-1}=(1/|A|)\text{ adj } A$.

14. ELEMENTARY TRANSFORMATIONS

Elementary row operations are: interchange of two rows, $R_i \leftrightarrow R_j$; multiplication of a row by a non-zero number, $R_i \rightarrow kR_i$; and addition of a multiple of one row to another, $R_i \rightarrow R_i + kR_j$. Similar operations can be performed on columns.

15. FINDING THE INVERSE USING ELEMENTARY OPERATIONS

Write the augmented matrix $[A \mid I]$, where I is the identity matrix of the same order. Apply elementary row operations until the left side becomes I. The right side then becomes A^{-1}. Thus, $[A \mid I] \rightarrow [I \mid A^{-1}]$.

Class 12 NIOS Mathematics – Matrices Formula Sheet

Topic	Formula / Condition	Important Point
Order of Matrix	$\boxed{m \times n}$	m = number of rows, n = number of columns
General Matrix	$\boxed{A = [a_{ij}]_{m\times n}}$	a_{ij} is the element in the i^{th} row and j^{th} column
Equality of Matrices	$\boxed{A = B \Rightarrow a_{ij} = b_{ij}}$	Both matrices must have the same order
Transpose	$\boxed{A^T}$	Rows become columns and columns become rows
Transpose Property	$\boxed{(A^T)^T = A}$	Transpose of transpose gives the original matrix
Transpose of Addition	$\boxed{(A+B)^T = A^T + B^T}$	Valid when addition is possible
Transpose of Subtraction	$\boxed{(A-B)^T = A^T - B^T}$	Valid when subtraction is possible
Transpose of Product	$\boxed{(AB)^T = B^T A^T}$	The order is reversed
Symmetric Matrix	$\boxed{A^T = A}$	$\boxed{a_{ij} = a_{ji}}$
Skew-Symmetric Matrix	$\boxed{A^T = -A}$	$\boxed{a_{ij} = -a_{ji}}$
Diagonal Elements of Skew-Symmetric Matrix	$\boxed{a_{ii} = 0}$	All main diagonal elements are zero
Matrix Addition	$\boxed{A + B = [a_{ij} + b_{ij}]}$	Both matrices must have the same order
Commutative Property of Addition	$\boxed{A + B = B + A}$	Addition is commutative
Associative Property of Addition	$\boxed{A + (B + C) = (A + B) + C}$	Addition is associative
Additive Identity	$\boxed{A + O = O + A = A}$	O is the zero matrix
Additive Inverse	$\boxed{A + (-A) = O}$	$-A$ is the additive inverse
Matrix Subtraction	$\boxed{A - B = [a_{ij} - b_{ij}]}$	Both matrices must have the same order
Subtraction Property	$\boxed{A - B \neq B - A}$	Matrix subtraction is generally not commutative
Scalar Multiplication	$\boxed{kA = [ka_{ij}]}$	Every element is multiplied by k
Scalar Distributive Property	$\boxed{k(A + B) = kA + kB}$	Scalar is distributed over addition
Addition of Scalars	$\boxed{(k + l)A = kA + lA}$	Both scalars are distributed
Scalar Associative Property	$\boxed{k(lA) = (kl)A}$	Scalars can be multiplied first
Condition for AB	$\boxed{\text{Columns of } A = \text{Rows of } B}$	Multiplication is possible only under this condition
Order of Product	$\boxed{(m \times n)(n \times p) = m \times p}$	Inner numbers must be equal; outer numbers give the answer

Matrix Multiplication	$\boxed{AB = [\sum a_{ik}b_{kj}]}$	Use the row-by-column method
Associative Property	$\boxed{(AB)C = A(BC)}$	Matrix multiplication is associative
Left Distributive Property	$\boxed{A(B + C) = AB + AC}$	Multiplication is distributed
Right Distributive Property	$\boxed{(A + B)C = AC + BC}$	Multiplication is distributed
Commutative Property	$\boxed{AB \neq BA}$	Matrix multiplication is generally not commutative
Identity Property	$\boxed{AI = IA = A}$	I is the identity matrix
Zero Product	$\boxed{A \neq O,\ B \neq O,\ AB = O}$	Two non-zero matrices can have a zero product
Inverse Property	$\boxed{AA^{-1} = A^{-1}A = I}$	A^{-1} is the inverse of A
Inverse Formula	$\boxed{A^{-1} = \frac{1}{\lvert A\rvert}\operatorname{adj} A}$	Applicable when $\lvert A\rvert \neq 0$
2×2 **Determinant**	(\boxed{\left	\begin{matrix}a&b\c&d\end{matrix}\right
Inverse of 2×2 **Matrix**	$\boxed{A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$	Condition: $\boxed{ad - bc \neq 0}$
Row Interchange	$\boxed{R_i \leftrightarrow R_j}$	Interchange two rows
Row Multiplication	$\boxed{R_i \to kR_i}$	$k \neq 0$
Row Replacement	$\boxed{R_i \to R_i + kR_j}$	Add a multiple of one row to another
Inverse by Elementary Operations	$\boxed{[A \mid I] \to [I \mid A^{-1}]}$	Convert the left side into the identity matrix

Most Important Formulas for Exam

$$\boxed{(A^T)^T = A}$$

$$\boxed{A^T = A}$$

$$\boxed{A^T = -A}$$

$$\boxed{(AB)^T = B^T A^T}$$

$$\boxed{(m \times n)(n \times p) = m \times p}$$

$$\boxed{AB \neq BA}$$

$$\boxed{AI = IA = A}$$

$$\boxed{AA^{-1} = A^{-1}A = I}$$

$$\boxed{A^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}}$$